# CONTINUOUS R.V

- continuous r.v **MUST** be defined over an interval of non-zero length

  [mathematics dealing with measure theory/integration]

- CDF of a *CONTINUOUS* r.v X is itself **CONTINUOUS**

  i $F_X(x \to x_+) = F_X(x \to x_-)\ ,\ \forall x$

- This is NOT true for discrete r.v : $F_X$ has jumps when P[X=x] is non-zero..



## CDF and PDF

- For a cont. r.v, the *probability distribution function (pdf)*

  $$f_X(x) = \frac{dF_X(x)}{dx}$$

- $$f_X(x) \geq 0\ ,\ \ \forall x$$
  $$F_X(x) = \int_{-\infty}^{x} f_X(u).du$$
  $$\int_{-\infty}^{\infty} f_X(u).du\ =\ 1$$



## Connection between P[.] and $f_X$

- P[x < X $\leq$ x+dx] = $f_X(x)$ dx
  - as well as its integral form

- E[X] in terms of pdf
  E[g(X)] in terms of pdf $f_X$

- Variance : Var[X] = E[$X^2$] - $\mu_X^2$

## Some important pdf's & cdf's

- uniform r.v over interval [a,b)
- exponential r.v - parameter *a*
- Erlang r.v - parameters *n* , $\lambda$
- Gaussian - parameters $\mu$ , $\sigma$
- Cauchy - parameters *a* , *b*
- Gamma - parameters *a* , *b*
- Laplace - parameters *a* , *b*
- Rayleigh - parameter *a*



## Guassian r.v

- Gaussian - parameters $\mu$ , $\sigma$
  E[X] = $\mu$ , Var[X] = $\sigma^2$
- Standard Normal r.v : $\mu = 0$ , $\sigma = 1$
  - Standard Normal cdf : $\Phi(z)$ is tabulated
- connection with general cdf with $\mu$ , $\sigma$ :

$$F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$



## Mixed r.v

- X is a mixed r.v $\Leftrightarrow$
  $f_X$ contains $\delta$-functions as well as non-zero finite values.
  - discrete r.v : $f_X$ contains ONLY $\delta$-functions

## Pdf's of derived r.v

- Given r.v X and its pdf $f_X(x)$, and the transformation to a new r.v Y by
  $Y=g(X)$
  then we determine the pdf $f_Y(y)$ by
  - determine the CFD $F_Y(y) = P[Y \leq y]$
  - determine the PDF $f_Y(y) = d\, F_Y(y)/dy$



## Conditional PDF

- The conditional PDF of X given a subset B of observations
  $f_{X|B}(x) = f_X(x)/P[B]$ , x in B
  $= 0$ , otherwise
- Compute conditional expectation values...